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GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

23. Proposed by E. L. PRATT, Tecumseh, Nebraska.

The ordinate of the point P of an ellipse is produced to meet the circle described on the major axis as diameter at Q . CQ , the straight line joining Q and the center of the ellipse, is tangent to the circle described on the focal radius of P as diameter.

If θ is the excentric angle prove that

$$\sin 2\theta = \frac{2(2a+b) \pm 4\sqrt{a(a+b)}}{a-b}.$$

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

I get a different result from that given in the problem by the following method:

Let (x', y') be the coordinates of P ; then the focal distance of P is $a - ex'$, and the half of it is the radius of the circle. The coordinates of the center of the circle are plainly $\frac{1}{2}(x' + ae)$, $\frac{y'}{2}$, and the equation to the circle is $\left(x - \frac{x' + ae}{2}\right)^2 + \left(y - \frac{y'}{2}\right)^2 = \left(\frac{a - ex'}{2}\right)^2 \dots (1)$, or, reducing, $x^2 + y^2 - (x' + ae)x - y'y + aex' = 0 \dots (2)$.

The coordinates of Q are $(x', \frac{a}{b}y')$ and the equation to CQ is $ay'x - bx'y = 0 \dots (3)$.

The condition that (3) touches (2) is given by $a^2y'^2 \left(aex' - \frac{y'^2}{4}\right) + b^2x'^2 \left(aex' - \frac{(x' + ae)^2}{4}\right) - \frac{1}{2}abx'y'^2(x' + ae) = 0 \dots (4)$, or $4a^3b^2ex' = [ay'^2 + bx'(x' + ae)]^2 \dots (5)$.

But $x' = a \cos \theta$, and $y' = b \sin \theta$; and (5) becomes $4a^4b^2e \cos \theta = [ab^2 \sin^2 \theta + ab \cos \theta(a \cos \theta + ae)]^2$ or $4a^2e \cos \theta = [b \sin^2 \theta + a \cos^2 \theta + ae \cos \theta]^2 \dots (6)$, a biquadratic in $\sin \theta$ or $\cos \theta$, consistent with the fact that there are four positions of P possible.

II. Solution by the PROPOSER.

Let (x', y') be the coordinates of the point P , and $\left(\frac{x_1 + e}{2}, \frac{y}{2}\right)$ the coordinates of the middle point of FP , the focal radius, where e is the linear excentricity of the ellipse. The equation of CQ is $y = x \tan \theta$. Employing the

usual formulas, $FP = \frac{a^2 - ex_1}{a}$; and the lengths of the perpendicular from the point $\left(\frac{x_1 + e}{2}, \frac{y_1}{2}\right)$ upon CQ is $\frac{y_1 - (x_1 + e) \tan \theta}{\pm 2\sqrt{1 + \tan^2 \theta}}$.

Since CQ is tangent to the circle upon FP this perpendicular $= \frac{FP}{2}$.

Therefore, substituting for (x_1, y_1) their values in terms of θ , we obtain,

$$(a-b)\sin \theta \cos \theta - a = -e(\sin \theta + \cos \theta).$$

Squaring and arranging $(a-b)^2 \sin^2 \theta \cos^2 \theta - 2(a-b)(2a+b) \sin \theta \cos \theta = -b^2$, which may be written $(a-b)^2 \sin^2 2\theta - 4(a-b)(2a+b) \sin 2\theta = -4b^2$.

$$\text{Solving, } \sin 2\theta = \frac{2(2a+b) \pm 4\sqrt{a(a+b)}}{a-b}.$$

Excellent solutions were also received from J. F. W. SCHEFFER, G. B. M. ZERR, and JOHN FAUGHT.

24. Proposed by T. W. PALMER, Professor of Mathematics in the University of Alabama.

Two right triangles have the same base, the hypotenuses of the first is equal to 60, of the second 40. The point of intersection of the two hypotenuses is at the distance 15 from the base. Find the length of the base.

Solution by SETH PRATT, C. E., Assyria, Michigan; A. H. BELL, Hillsboro, Illinois; and P. S. BERG, Apple Creek, Ohio.

Let $AD = x - y$; $BC = x + y$; $EH = c = 15$; $BD = b = 40$; $AC = a = 60$; $AF = z$; and $FB = w$. By similar triangles, $w : c = w + z : x - y$, or $w(x - y) = c(w + z) \dots (1)$; and $z : c = w + z : x + y$, or $(w + z)c = z(x + y) \dots (2)$.

$\therefore w(x - y) = z(x + y) \dots (3)$. Put $x + y = s$ and $x - y = t$. Then $wt = zs$ or $w = zs/t \dots (4)$. Substituting this value of w in the proportion of (2), $\frac{2s}{t} : c$

$= \frac{zt + zs}{t} : t$. Dividing antecedents by $\frac{z}{t}$, we have $s : c$

$$= t + s : t, \text{ or } st = c(t + s), \text{ whence } c = \frac{st}{s + t} = \frac{(x + y)(x - y)}{x + y + x - y} = \frac{x^2 - y^2}{2x} \dots (5).$$

From (5), $x^2 - 2cx = y^2$, whence $x = c + \sqrt{y^2 + c^2}$. From the figure, $AC^2 - BC^2 = AB^2$ and $BD^2 - AD^2 = AB^2$. $\therefore AC^2 - BC^2 = BD^2 - AD^2$, or $a^2 - (x + y)^2 = b^2 - (x - y)^2$, whence $x = \frac{a^2 - b^2}{4y}$. Equating the two values of x , $\frac{a^2 - b^2}{4y} = c + \sqrt{y^2 + c^2}$.

From this, we have $y^4 + \frac{1}{2}c(a^2 - b^2)y = (a^2 - b^2)^2$.

Restoring numbers, and solving the resulting equation, $y = 14.060811$.

*The sequence is, that AB may be any number whatever when AC and BD are indefinite.

